# GCE A LEVEL MARKING SCHEME

## SUMMER 2023

## A LEVEL
## FURTHER MATHEMATICS
## UNIT 6 FURTHER MECHANICS B
## 1305U60-1

## INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

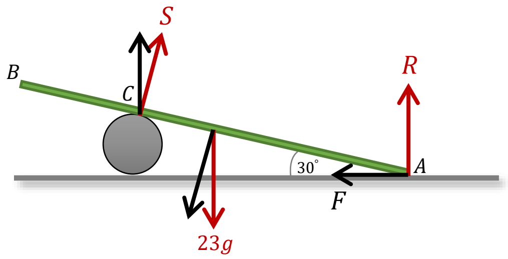
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

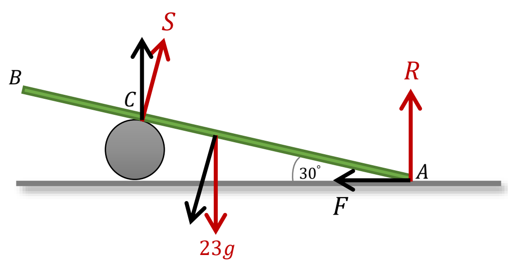
# WJEC GCE A LEVEL FURTHER MATHEMATICS

## UNIT 6 FURTHER MECHANICS B

## SUMMER 2023 MARK SCHEME

| Q1 | Solution | Mark | Notes |
|---|---|---|---|
| (a) |  | | |
| | Resolve vertically | M1 | Dim. correct equation with 3 terms |
| | $S\cos 30° + R = 23g$ $\quad \frac{\sqrt{3}}{2}S + R = 23g$ | A1 | $23g = 225\cdot 4 = \frac{1127}{5}$ |
| | Resolve horizontally | M1 | Dim. correct equation |
| | $F = S\sin 30°$ $\quad F = \frac{1}{2}S$ | A1 | |
| | $F = \frac{2}{3}R$<br>$\frac{2}{3}R = S\sin 30°$ $\quad \frac{2}{3}R = \frac{1}{2}S$<br>$R = \frac{3}{4}S$ | B1 | Used |
| | $\frac{\sqrt{3}}{2}S + \frac{3}{4}S = 23g$ | m1 | Elimination of one variable<br>Both M's needed above |
| | $S = 139\cdot 47(80054)$ (N)<br>$R = 104\cdot 60(85041)$ (N) | A1<br>A1 | cao<br>cao |
| | Note exact forms | **[8]** | |
| | $S = \frac{92g}{3}(2\sqrt{3}-3) = \frac{92g}{2\sqrt{3}+3}$ | | |
| | $R = 23g(2\sqrt{3}-3) = \frac{69g}{2\sqrt{3}+3}$ | | |
| (b) | Moments about $A$ | M1 | Dim. correct equation, no extra terms |
| | $AC \times S = 23g \times 4\cos 30°$ | A1<br>A1 | LHS<br>RHS |
| | $AC \times 139\cdot 478 ... = 225\cdot 4 \times 2\sqrt{3}$ | | |
| | $AC = \frac{23g \times 4\cos 30°}{139\cdot 4780054}$ $\quad \frac{225\cdot 4 \times 2\sqrt{3}}{139\cdot 4780054}$ | | FT $S$ from (a) |
| | $AC = 5\cdot 598076211$ (m) or $= \frac{6+3\sqrt{3}}{2}$ (m) | A1 | cao |
| | | **[4]** | |

| | | | |
|---|---|---|---|
| (c) | No, both reactions remain the same<br>**and** (one of the following)<br>• Calculations in (a) are **independent** of the location of the centre of mass<br>• **Moments** were **not used** in part (a) and so distances were not considered | E1<br><br>**[1]** | |
| | Total for Question 1 | 13 | |

| Q1 | Alternative Solution | Mark | Notes |
|---|---|---|---|
| (a) |  | | |
| | Resolve parallel to $AB$ | M1 | Dim. correct equation with 3 terms |
| | $23g \sin 30° = R \sin 30° + F \cos 30°$ | A1 | $23g \sin 30° = \frac{23}{2}g = \frac{1127}{10} = 112 \cdot 7$ |
| | $\frac{23}{2}g = \frac{1}{2}R + \frac{\sqrt{3}}{2}F \quad \Leftrightarrow \quad 23g = R + \sqrt{3}F$ | | |
| | Resolve perpendicular to $AB$ | M1 | Dim. correct equation with 4 terms |
| | $S + R \cos 30° = 23g \cos 30° + F \sin 30°$ | A1 | |
| | $S + \frac{\sqrt{3}}{2}R = \frac{23\sqrt{3}}{2}g + \frac{1}{2}F \Leftrightarrow 2S + \sqrt{3}R = 23\sqrt{3}g + F$ | | |
| | $F = \frac{2}{3}R$ | B1 | Used |
| | Elimination of one variable (in either equation) | m1 | |
| | $23g = R + \sqrt{3}(\frac{2}{3}R)$ | | |
| | $S = 139 \cdot 47(80054)$ (N)<br>$R = 104 \cdot 60(85041)$ (N) | A1<br>A1 | cao<br>cao |
| | | **[8]** | |
| | Note exact forms<br>$$S = \frac{92g}{3}(2\sqrt{3} - 3) = \frac{92g}{2\sqrt{3} + 3}$$<br>$$R = 23g(2\sqrt{3} - 3) = \frac{69g}{2\sqrt{3} + 3}$$ | | |

| Q2 | Solution | Mark | Notes |
|---|---|---|---|
| | | | |

| Shape | Volume/Mass | Distance of COM from base | Mark | Notes |
|---|---|---|---|---|
| Large Cone | $\frac{1}{3}\pi(3x)^2(6y)\rho$<br>$(= 18\pi x^2 y)$ | $\frac{1}{4}(6y)$<br>$\left(= \frac{3y}{2}\right)$ | B1 | Condone omission of $\rho$ (mass per unit volume)<br>Both volume and distance |
| Small Cone | $\frac{1}{3}\pi(x)^2(2y)\rho$<br>$\left(= \frac{2}{3}\pi x^2 y\right)$ | $4y + \frac{1}{4}(2y)$<br>$\left(= \frac{9y}{2}\right)$ | B1<br>B1 | Identification of $2y$ in either<br>For both volume and distance |
| Frustum | $\left(18\pi x^2 y - \frac{2}{3}\pi x^2 y\right)\rho$<br>$\left(= \frac{52}{3}\pi x^2 y\right)$ | $\bar{y}$ | B1 | cao |

| Solution | Mark | Notes |
|---|---|---|
| Moments about base | M1 | Masses and moments consistent<br>All terms, allow sign errors<br>FT table throughout |
| $\frac{52}{3}\pi x^2 y \times \bar{y} = 18\pi x^2 y \times \frac{3y}{2} - \frac{2}{3}\pi x^2 y \times \frac{9y}{2}$ | A1 | |
| $\frac{52}{3} \times \bar{y} = 18 \times \frac{3y}{2} - \frac{2}{3} \times \frac{9y}{2}$ | | |
| $\bar{y} = \frac{18}{13}y$ | A1<br>[7] | Convincing (cao) |
| Total for Question 2 | 7 | |

| Q2 | Alternative Solution | Mark | Notes |
|---|---|---|---|
| (b) | | | Condone omission of $\rho$ (mass per unit volume) |

| Shape | Volume/Mass | Distance of COM from vertex |
|---|---|---|
| Large Cone | $\frac{1}{3}\pi(3x)^2(6y)\rho$ <br> $(= 18\pi x^2 y)$ | $\frac{3}{4}(6y)$ <br> $\left(=\frac{9y}{2}\right)$ |
| Small Cone | $\frac{1}{3}\pi(x)^2(2y)\rho$ <br> $\left(=\frac{2}{3}\pi x^2 y\right)$ | $\frac{3}{4}(2y)$ <br> $\left(=\frac{3y}{2}\right)$ |
| Frustum | $\left(18\pi x^2 y - \frac{2}{3}\pi x^2 y\right)\rho$ <br> $\left(=\frac{52}{3}\pi x^2 y\right)$ | $\bar{y}$ |

| | Working | Mark | Notes |
|---|---|---|---|
| | Large Cone row | B1 | Both volume and distance |
| | Small Cone row | B1 | Identification of $2y$ in either |
| | | B1 | Both volume and distance |
| | Frustum row | B1 | cao |
| | Moments about vertex <br> $\frac{52}{3}\pi x^2 y \times \bar{y} = 18\pi x^2 y \times \frac{9y}{2} - \frac{2}{3}\pi x^2 y \times \frac{3y}{2}$ | M1 <br> A1 | Masses and moments consistent <br> All terms, allow sign errors <br> FT table throughout |
| | $\frac{52}{3} \times \bar{y} = 18 \times \frac{9y}{2} - \frac{2}{3} \times \frac{3y}{2}$ | | |
| | $\bar{y} = \frac{60}{13}y$ | | |
| | $\therefore$ Distance from base $= 6y - \frac{60}{13}y = \frac{18}{13}y$ | A1 <br> **[7]** | Convincing (cao) |

| | | | Condone omission of $\rho$ (mass per unit volume) |
|---|---|---|---|

| Shape | Volume/Mass | Distance of COM from top of frustum |
|---|---|---|
| Large Cone | $\frac{1}{3}\pi(3x)^2(6y)\rho$ <br> $(= 18\pi x^2 y)$ | $\frac{3}{4}(6y) - 2y$ <br> $\left(= \pm\frac{5y}{2}\right)$ |
| Small Cone | $\frac{1}{3}\pi(x)^2(2y)\rho$ <br> $\left(=\frac{2}{3}\pi x^2 y\right)$ | $\frac{1}{4}(2y)$ <br> $\left(= \pm\frac{y}{2}\right)$ |
| Frustum | $\left(18\pi x^2 y - \frac{2}{3}\pi x^2 y\right)\rho$ <br> $\left(=\frac{52}{3}\pi x^2 y\right)$ | $\bar{y}$ |

| | Working | Mark | Notes |
|---|---|---|---|
| | Large Cone row | B1 | Sight of $2y$ in either |
| | Small Cone row | B1 | Identification of $2y$ in either |
| | | B1 | Both volume and distance |
| | Frustum row | B1 | cao |
| | Moments about smaller circular surface <br> $\frac{52}{3}\pi x^2 y \times \bar{y} = 18\pi x^2 y \times \pm\frac{5y}{2} - \frac{2}{3}\pi x^2 y \times \mp\frac{y}{2}$ | M1 <br> A1 | Masses and moments consistent <br> All terms, allow sign errors <br> FT table throughout |
| | $\frac{52}{3} \times \bar{y} = 18 \times \frac{5y}{2} - \frac{2}{3} \times -\frac{y}{2}$ | | |
| | $\bar{y} = \frac{34}{13}y$ | | |
| | $\therefore$ Distance from base $= 4y - \frac{34}{13}y = \frac{18}{13}y$ | A1 <br> **[7]** | Convincing (cao) |

| Q3 | Solution | Mark | Notes |
|---|---|---|---|
| (a) | period $= 12$ (hours) $\quad T = 12$ | B1 | si |
| | amplitude $= 4$ (m) $\quad a = 4$ | B1 | si |
| | | [2] | |
| (b) | $\omega = \frac{\pi}{6}$ $\quad \omega = \frac{\pi}{21600}$ | B1 | $\omega = \frac{2\pi}{\text{period}}$ FT period from (a) |
| | Using $x = \pm a\cos(\omega t)$ | M1 | Allow $\pm a\sin(\omega t)$ |
| | $x = -4\cos\left(\frac{\pi}{6}t\right)$ | A1 | oe, cao |
| | | [3] | |
| (c) | $-2 = -4\cos\left(\frac{\pi}{6}t\right)$ | M1 | FT $x$ from (b) |
| | $t = 2$ (hours) | A1 | cao |
| | $t = 10$ (hours) | A1 | cao |
| | Earliest time: 7 a.m. and Latest time: 3 p.m. | A1 | cao, both times |
| | | [4] | |
| (d) | $v = \frac{dx}{dt}$ | M1 | FT $x$ from (b) |
| | $v = 4\sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6}$ | A1 | |
| | $v = \frac{2}{3}\pi\sin\left(\frac{\pi}{6}t\right)$ | | |
| | At $t = 9$, $v = \frac{2}{3}\pi\sin\left(\frac{\pi}{6} \times 9\right)$ | M1 | FT $v$ |
| | $v = -\frac{2}{3}\pi \quad = -2\cdot 0(94\ldots)$ | | |
| | Rate (at which the level of water is falling) $= \frac{2}{3}\pi$ (m/hour) | A1 | |
| | | [4] | |
| | Total for Question 3 | 13 | |

| Further Notes | |
|---|---|
| (b) | Equivalent forms for A1 |
| | $x = -4\cos\left(\frac{\pi}{6}t\right) \quad = 4\sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) \quad = -4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) \quad = 4\cos\left(\frac{\pi}{6}t \pm \pi\right)$ |
| | Corresponding derivatives for part (d) |
| | $v = x' = 4\sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} \quad = 4\cos\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) \times \frac{\pi}{6} \quad = -4\cos\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) \times \frac{\pi}{6} \quad = -4\sin\left(\frac{\pi}{6}t \pm \pi\right) \times \frac{\pi}{6}$ |

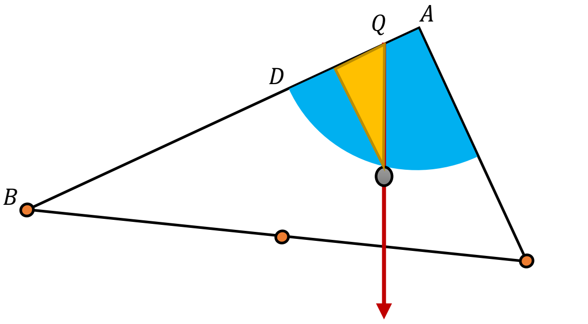
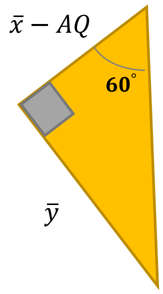
| Q3 | Alternative Solution | Mark | Notes |
|---|---|---|---|
| | Using $t = 9$ $(5\text{ a.m. to } 2\text{ p.m.})$ to deduce that | M1 | $x = -4\cos\left(\frac{\pi}{6} \times 9\right)$ |
| | $x = 0$ | A1 | |
| | Using an expression for $v$ with $x = 0$ | m1 | |
| | $v = \pm\begin{cases}\omega\sqrt{a^2 - 0^2} \\ \omega a\end{cases}$ | | |
| | $v = \pm\frac{2}{3}\pi$ $\quad (= \pm 2\cdot 0(94 \ldots))$ | | |
| | Rate (at which the level of water is falling) $= \frac{2}{3}\pi \quad (\text{m/hour})$ | A1 | |
| | | **[4]** | |

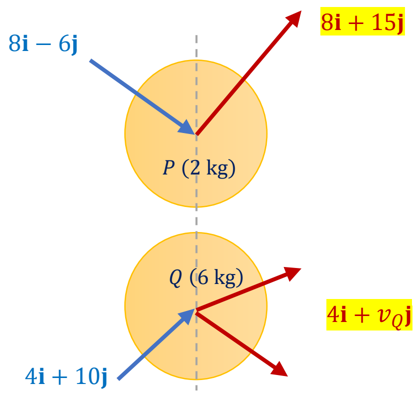
| Q4 | Solution | Mark | Notes |
|---|---|---|---|
| (a) | (see table below) | | |

| Shape | Mass | Distance from AC | Distance from AB | Mark | Notes |
|---|---|---|---|---|---|
|  | $\frac{\pi(12)^2}{4} \times 2m$ $(= 72m\pi)$ | $\frac{16}{\pi}$ | $\frac{16}{\pi}$ | B1<br>B1 | Correct mass<br>Both distances correct |
| $C$ | $50m$ | $0$ | $28$ | | |
| $F$ | $30m$ | $22 \cdot 5$ | $14$ | B2 | Masses and distances for C, F, B |
| $B$ | $20m$ | $45$ | $0$ | | |
| *Lamina* | $(72\pi + 100)m$ | $\bar{x}$ | $\bar{y}$ | B1 | cao |

| Solution | Mark | Notes |
|---|---|---|
| (i) Moments about AC | M1 | Masses and moments consistent<br>All terms, allow sign errors<br>FT table |
| $\left((72\pi)\left(\frac{16}{\pi}\right) + (30)(22 \cdot 5) + (20)(45)\right)m = (72\pi + 100)m \times \bar{x}$ | A1 | |
| $(1152 + 675 + 900)m = (72\pi + 100)m \times \bar{x}$ | | |
| $\bar{x} = 8 \cdot 36(0038474)$ (cm) | A1 | cao |
| (ii) Moments about AB | M1 | Masses and moments consistent<br>All terms, allow sign errors<br>FT table |
| $\left((72\pi)\left(\frac{16}{\pi}\right) + (30)(14) + (50)(28)\right)m = (72\pi + 100)m \times \bar{y}$ | A1 | |
| $(1152 + 420 + 1400)m = (72\pi + 100)m \times \bar{y}$ | | |
| $\bar{y} = 9.11(1123705)$ (cm) | A1<br>**[11]** | cao |

| Q4 | Solution | Mark | Notes |
|---|---|---|---|
| (b) | Length $AP = \bar{y}$ | B1<br>**[1]** | FT $\bar{y}$ from (a)(ii) |

| | | | |
|---|---|---|---|
| (c) |  | |  |
| | If hanging in equilibrium, vertical passes through centre of mass. | M1 | Correct triangle identified<br>FT $\bar{x}$ and $\bar{y}$ from (a) |
| | $\tan 60° = \dfrac{\bar{y}}{\bar{x} - AQ}$ $\qquad \sqrt{3} = \dfrac{9.11\ ...}{8\cdot36\ ... - AQ}$ | A1 | FT $\bar{x}$ and $\bar{y}$ from (a)<br>$\bar{x} - AQ = \dfrac{\bar{y}}{\tan 60°} = \bar{y}\cot 60°$<br>$= 5\cdot2603\ ...$ |
| | $AQ = \bar{x} - \frac{\bar{y}}{\sqrt{3}}$ | | |
| | $AQ = 3.0997\ .....$ (cm) | A1 | FT $\bar{x}$ and $\bar{y}$ from (a) provided $AQ < 12$, i.e. $Q$ lies on $AD$ |
| | | [3] | |
| | Total for Question 4 | 15 | |

| Q5 | Solution | Mark | Notes |
|---|---|---|---|
| (a) | $\mathbf{r}_P = (8\mathbf{i} - 6\mathbf{j})t$<br>$\mathbf{r}_Q = (12\mathbf{i} - 48\mathbf{j}) + (4\mathbf{i} + 10\mathbf{j})t$ | B1<br>B1 | |
| | If spheres collide, then $\mathbf{r}_P = \mathbf{r}_Q$ for some value of $t$.<br>Comparison of coefficients | M1 | Both i and j coefficients compared |
| | $\mathbf{i}$ $\quad 8t = 12 + 4t$<br>$t = 3$<br><br>$\mathbf{j}$ $\quad -6t = -48 + 10t$<br>$t = 3$ | | |
| | Value of $t$ for both components are equal, therefore spheres collide. | A1<br><br>**[4]** | |
| (b) |  | | Before collision<br><br>After collision |
| | Speed $= 5$<br>$\sqrt{4^2 + {v_Q}^2} = 5$ | M1 | Used |
| | $v_Q = \pm 3$ | A1 | At least one value |
| | Con. of momentum (along line of centres) | M1 | Attempted |
| | $(2)(-6) + (6)(10) = 2v_P + 6v_Q$<br>$48 = 2v_P + (6)(\pm 3)$ | A1 | All correct. FT $v_Q$ |
| | $v_Q = +3 \quad \Rightarrow \quad v_P = 15$<br>$v_Q = -3 \quad \Rightarrow \quad v_P = 33$ | A1 | Both values of $v_P$ |
| | Restitution (along line of centres)<br>$v_Q - v_P = -e(10 - -6)$ | M1 | FT $v_Q$ and $v_P$ |
| | $3 - 15 = -e(10 - -6)$ OR $-3 - 33 = -e(10 - -6)$<br>$e = \frac{3}{4}$ $\qquad e = \frac{9}{4}$ | A1 | All correct with their $v_P$ corresponding to either $v_Q$ |
| | $e = \frac{3}{4}$ | A1 | Both values of $e$ with $e\left(= \frac{9}{4}\right) > 1$ clearly discarded |
| | Velocity of sphere $P$, $\mathbf{v}_P = 8\mathbf{i} + 15\mathbf{j}$ $\quad (\text{ms}^{-1})$ | A1<br><br>**[9]** | cao |

| | | | |
|---|---|---|---|
| (c) | Impulse, $\mathbf{I}$ = change in momentum<br>$\mathbf{I} = 2(0 - (8\mathbf{i} + 15\mathbf{j}))$<br>$\mathbf{I} = -(16\mathbf{i} + 30\mathbf{j})$<br>$\lvert\mathbf{I}\rvert = 34 \ \ (\text{Ns})$ | M1<br><br>A1<br>A1<br>**[3]** | Used<br><br>FT $\mathbf{v}_P$<br>cao |
| | Total for Question 5 | **16** | |

| Q5 | Alternative Solutions | Mark | Notes |
|---|---|---|---|
| (a) | $\mathbf{r}_Q - \mathbf{r}_P = (12 - 4t)\mathbf{i} + (-48 + 16t)\mathbf{j}$<br>$\mathbf{r}_Q - \mathbf{r}_P = 0$<br>$\Rightarrow \quad 12 - 4t = 0 \quad \text{and} \quad -48 + 16t = 0$<br>$t = 3$ for both components are equal, therefore spheres collide. | B1<br>B1<br><br>M1<br>A1<br>**[4]** | B1 for each component |
| (a) | $\mathbf{r}_Q - \mathbf{r}_P = (12 - 4t)\mathbf{i} + (-48 + 16t)\mathbf{j}$<br>Forming and solving a quadratic equation<br>$\lvert\mathbf{r}_Q - \mathbf{r}_P\rvert^2 = 272t^2 - 1632t + 2448 = 0$<br>$t = 3$ or $b^2 - 4ac = 0$, therefore spheres collide. | B1<br>B1<br><br>M1<br>A1<br>**[4]** | B1 for each component |

| Q6 | Solution | Mark | Notes |
|---|---|---|---|
| (a) | Application of N2L | M1 | Dim. correct. $250\,000v$ and $T$ in same direction |
| | $-T - 250\,000v = 50\,000a$ | A1 | |
| | $T = 312\,500x$ | B1 | $a = \frac{d^2x}{dt^2}$ |
| | $-312\,500x - 250\,000v = 50\,000\frac{d^2x}{dt^2}$ | | $\div 12\,500$ |
| | $4\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 25x = 0$ | A1 | Convincing |
| | | **[4]** | |
| (b) | Auxiliary equation: $4r^2 + 20r + 25 = 0$<br>Roots: $r = -\frac{5}{2}$ (twice) | M1<br>A1 | |
| | General solution: $x = e^{-\frac{5}{2}t}(At + B)$ | B1 | |
| | Initial conditions $t = 0,\ x = 0,\ (x' = U)$ | M1 | Used in general solution |
| | $B = 0 \qquad U = -\frac{5}{2}B + A$ | A1 | cao |
| | Differentiating<br>$v = x' = -\frac{5}{2}e^{-\frac{5}{2}t}(At + B) + e^{-\frac{5}{2}t}A$ | M1<br>A1 | $U = -\frac{5}{2}B + A$<br>Initial conditions give $A = U$ |
| | $\therefore\ x = Ue^{-\frac{5}{2}t}t$ | A1 | cao |
| | | **[8]** | |
| (c) | $v = x' = Ue^{-\frac{5}{2}t}\left(1 - \frac{5}{2}t\right)$ | | |
| | When $v = 0$ $\left(\Rightarrow\ t = \frac{2}{5}\right)$ | M1 | FT $v = x'$ |
| | Using $x = Ue^{-\frac{5}{2}t}t$ at $t = \frac{2}{5}$ and $x = 0\cdot3$. | m1 | FT $v = x'$ and $t > 0$ |
| | $0\cdot3 = Ue^{-\frac{5}{2}\left(\frac{2}{5}\right)}\left(\frac{2}{5}\right)$ | | |
| | $U = \frac{3}{4}e = 2.0387\ldots \qquad (\text{ms}^{-1})$ | A1 | cao |
| | | **[3]** | |
| (d) | Critical damping<br>Repeated root in (b) | E1<br><br>**[1]** | oe, $b^2 - 4ac = 0$<br>**or**<br>$k^2 - \omega^2 = 0$ when written in the form<br>$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2x = 0$ |
| | Total for Question 6 | **16** | |